Math 112 Discrete Mathematics

Exercises 10

1) In a playoff series, the probability that Team A wins over Team B is 3/5 and the probability that Team C wins over Team D is 4/7. Find the probability that Team A wins and Team C loses.

2) Three checkers are located in the cells of a 8×8 checkerboard (at most one checker in each cell). Describe the sample space and compute the probability that no row or column contains more than one checker.

3) Two fair six-sided dice are rolled and the face values are added. Find the probability of obtaining an odd number greater than 8.

4) Three cards are pulled from a deck of 52 cards. Find the probability of obtaining at least one clubs.

5) If a fair six-sided die is tossed twice, find the probability that the first toss will be a number less than 4 and the second toss will be a number greater than 4.

6) Two cards are drawn without replacement from a deck of 52 cards. Find the probability of the first card being a red face card and the second card being a clubs.

7) Two dice are thrown. What is the probability that the product of the facing numbers is a prime?

8) Three different DVD's and their corresponding DVD cases are randomly strewn about on a shelf. If a child puts the DVD's in the cases at random, find the probability of correctly matching all DVD's and cases.

9) Derya is looking for a top-floor apartment. She hears about two vacant apartments in a building with 7 floors and 8 apartments per floor. What is the probability that there is a vacant apartment on the top floor?

10) You choose at random two cards from a standard deck of 52 cards. A card is called a winning card if it is a ten or/and a hearts. What is the probability of getting a winning card? If two cards are drawn at once, what is the probability of having at least one winning card?

11) A monkey at a typewriter types each of the 26 letters of the alphabet exactly once, the order being random. What is the probability that the word 'MONKEY' appears somewhere in the string of letters?

12) You draw at random five cards from a standard deck of 52 cards. What is the probability that there is an ace among the five cards and a king or queen?

13) A box contains 7 apples and 5 oranges. The fruits are taken out of the box, one at a time and in a random order. What is the probability that the bowl will be empty after the last apple is taken from the box?

14) A group of five people simultaneously enter an elevator at the ground floor. There are 10 upper floors. They choose their exit floors independently of each other.

a) Describe a sample space and determine the probability that they are all going to different floors.

b) How does the answer change when each person chooses with probability 1/2 the 10th floor as the exit floor and the other floors remain equally likely as the exit floor with a probability of 1/18 each?

15) Three balls are randomly dropped into three boxes, where any ball is equally likely to fall into each box. Specify an appropriate sample space and determine the probability that exactly one box will be empty.

16) Three couples attend a dinner. Each of the six people chooses randomly a seat at a round table. What is the probability that no couple sits together?

17) Three friends and seven other people are randomly seated in a row. What is the probability that the three friends will sit next to each other?

18) In a group of n boys and n girls, each boy chooses at random a girl and each girl chooses at random a boy. The choices of the boys and girls are independent of each other. If a boy and a girl have chosen each other, they form a couple. What is the probability that no couple will be formed?

19) Six married couples participate in a tournament. The group of 12 people is randomly split into eight teams of three people each, where all possible splits are equally likely. What is the probability that none of the teams has a married couple?

20) In a high school class, 35% of the students take Spanish as a foreign language, 15% take French as a foreign language, and 40% take at least one of these languages. What is the probability that a randomly chosen student takes French given that the student takes Spanish?

21) A bowl contains four red and four blue balls. As part of drawing lots, you choose four times two balls at random from the bowl without replacement. What is the probability that one red and one blue ball are chosen each time?

22) In a restaurant, two of five people ordered coffee and the others ordered tea. The waiter forgot who ordered what and put the drinks in a random order. Describe the sample space and find the probability that each one gets the correct drink

23) A parking lot has 10 parking spaces arranged in a row. There are 7 cars parked. Assume that each car owner has picked at a random a parking place among the spaces available. Specify an appropriate sample space and determine the probability that the three empty places are adjacent to each other.

24) You and two of your friends are in a group of 10 people. The group is randomly split up into two groups of 5 people each. Specify an appropriate sample space and determine the probability that you and your two friends are in the same group.

25) Each student in a group of 10 has drawn an integer from {1, 2,3,4}. What is the probability that at least one of the integers is not picked by anyone?

26) There are three English teams among the eight teams that have reached the quarter-finals of the Champions League soccer. What is the probability that the three English teams will avoid each other in the draw if the teams are paired randomly?

27) A jar contains three white balls and two black balls. Each time you pick at random one ball from the jar. If it is a white ball, a black ball is inserted instead; otherwise, a white ball is inserted instead. You continue until all balls in the jar are black. What is the probability that you need no more than five picks to achieve this?

28) Each of hundred students chooses at random a number from $\{1, 2, \ldots, 9\}$, independently of the others. Next the chosen numbers are announced one by one. The first student who announces a number that has been announced before wins a bonus. Which person has the largest probability to win the bonus?

29) In a game with three players X, Y, Z the winner is chosen by the following procedure. A deck consisting of 1 black and 8 white cards is shuffled and in the order X, Y, Z, X, Y… a card from the deck is dealt to each player until someone gets the black card. First player receiving the black card is the winner.

a) What are the chances of players to win?

b) What are the chances of players if the deck consists of 2 black and 7 white cards?

30) Selim and Baran play a series of games until one of the players has won two games more than the other player. Any game is won by Selim with probability p. The results of the games are independent of each other. Assuming that Selim is the first player, what is the probability that he wins the match?