Math 112 Discrete Mathematics

Exercises 11

1) Five percent of the workers at a given factory are women. Suppose 10 workers are selected at random to be interviewed about quality of work conditions.
   a) What is the probability that two of the workers will be women?
   b) What is the probability that at least two of the workers will be women?
   c) What is the probability that none will be women?

2) A die is rolled until a five appears,
   a) what is the probability that the first five appears on the first trial?
   b) what is the probability that the first five appears on the 2nd trial?
   c) what is the probability that the first five appears before the 3rd trial?
   d) what is the probability that the first five appears on or before the 10th trial?
   e) what is the probability that the first five appears at the k-th trial?
   f) what is the probability that the first five appears after the 5th trial?

3) If a five-letter word is formed at random (meaning that all sequences of five letters are equally likely), what is the probability that no letter occurs more than once?

4) A consumer organization estimates that over a 1-year period 17% of cars will need to be repaired once, 7% need repairs twice, and 4% will require three or more repairs. If you own two cars, what is the probability that
   a) neither will need repair?
   b) both will need repair?

5) A slot machine has three wheels that spin independently. Each has 10 equally likely symbols: 4 bars, 3 lemons, 2 cherries, and a bell. If you play, what is the probability
   a) you get 3 lemons?
   b) you get no fruit symbols?
   c) you get 3 bells (the jackpot)?
   d) you get no bells?
   e) you get at least one bar (automatically lose)?

6) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople.
   a) What is the likelihood that 5 will be broken?
   b) What is the likelihood that they will all work?
   c) What is the likelihood that they will all be broken?

7) A KONIA cell phone is made from 224 components. Each component has a .001 probability of being defective. What is the probability that a KONIA cell phone will not work perfectly?

8) A company manufactures toy robots. About 3 toy robots per 100 does not work. You purchase 35 toy robots. What is the probability that exactly 4 do not work?

9) The πR-50 Company manufactures tires. They claim that only .007 of πR-50 tires are defective. What is the probability of finding a defective tires in a set of 5 tires? What is the probability of finding at least four non-defective tires in a set of 5 tires?

10) A farmer plants 12 saplings. On average, 15% of saplings planted fail to survive their first winter. Find the probability that more than one of his saplings will die in that first winter.

11) A fair die is rolled six times. What is the probability that the largest number rolled is r for $r = 1, ..., 6$?

12) A door has 3 locks on it. The door is opened if at least 2 of these locks are unlocked. In a box there are 10 keys, three of which are the keys of the locks on the door. If 3 keys are picked at random, what is the probability of opening the door?

13) A box contains 1 black and 4 white balls. A boy picks a ball at random and if that ball is black he stops. Otherwise he returns the ball in the box and repeats the process. Find the probability that the boy stops until the k th try.

14) A box contains 1 black and 2 white balls. A boy picks a ball at random and if that ball is black he stops. Otherwise he returns the ball together with a new white ball in the box and repeats the process. Find the probability that the boy stops until the k th try.

15) Bill and Mark take turns picking a ball at random from a bag containing four red balls and seven white balls. The balls are drawn out of the bag without replacement and Mark is the first person to start. What is the probability that Bill is the first person to pick a red ball?

16) The probability of rain on any given day in April in Rize is 0.8. Assuming that the weather on each day is independent of the weather on other days, find the probability that it rains on at least 20 days in April.

17) At a certain intersection, the traffic light is red for 30 seconds, yellow for 5 seconds, and green for 45 seconds. Find the probability that out of the next eight cars that arrive randomly at the light, exactly three will be stopped by a red light.

18) A baker put 500 raisins into dough, mixed well, and made 100 cookies. You take a random cookie. What is the probability of finding at least 4 raisins in it?

19) A biased coin is flipped 6 times. Show that the probability of having three Heads and three tails cannot be equal to 1/3.

20) When a biased coin is flipped 4 times, the probability of two heads and two tails is 0.24. Compute the probability of having an equal number of heads and tails when the same coin is flipped 6 times.

21) You flip a coin either 10 times or 20 times and win the game if the number of heads is equal to the number of tails. Which one is better, flipping 10 times or 20 times?