

METU  
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

**Exercises 12**

- 1) A couple has two children and the older child is a boy. What is the probability that the couple has two boys?
- 2) A couple has two children, one of which is a boy. What is the probability that the couple has two boys?
- 3) Box  $B_1$  contains 2 white and 3 red balls; box  $B_2$  contains 6 white and 4 red balls. One of the boxes is chosen randomly and from this box a ball is picked, again randomly. If the ball is white, what is the probability that the chosen box is  $B_1$ .
- 4) Box  $B_1$  contains 2 white and 3 red balls; box  $B_2$  contains 6 white and 4 red balls. All the balls are put in a bag. Then, one of the balls is picked randomly. If the ball is white, what is the probability that it is a ball from box  $B_1$ .
- 5) A box contains 2 green, 4 yellow and 6 white balls; another box contains 15 green, 10 yellow and 5 white balls. One of the boxes is chosen randomly and from that a box a ball is picked. Denote the event of choosing the first and the second boxes by I and II, respectively and let  $G, Y, W$  stand for the events of picking a green, a yellow and a white ball, respectively. Determine whether each of the events  $G, Y$  and  $B$  is independent with the event I or not.
- 6) Three fair dice are rolled. What is the probability that the sum of the three outcomes is 10 given that the three dice show different outcomes?
- 7) A bag contains four balls. One is blue, one is white and two are red. Someone draws together two balls at random from the bag. He looks at the balls and tells you that there is a red ball among the two balls drawn out. What is the probability the other ball drawn out is also red?
- 8) You pick an entry of a  $5 \times 10$  matrix randomly. Let  $A$  be the event that the entry comes from an odd-numbered row and  $B$  be the event that the entry comes from the first five columns. Are the events  $A$  and  $B$  independent?
- 9) Your friend has chosen at random a card from a standard deck of 52 cards but keeps this card concealed. You have to guess what card it is. Before doing so, you can ask your friend either the question whether the chosen card is red or the question whether the card is the ace of spades. Your friend will answer truthfully. What question would you ask?
- 10) Suppose that the probability of living to be older than 70 is 0.6 and the probability of living to be older than 80 is 0.2. If a person reaches her 70<sup>th</sup> birthday, what is the probability that she will celebrate her 80<sup>th</sup>?
- 11) A hat contains a number of cards, with 30% white on both sides, 50% black on one side and white on the other, 20% black on both sides. The cards are mixed up, then a single card is drawn at random and placed on the table. If the top side is black, what is the chance that the other side is white?
- 12) Experience shows that 95% of people buying airline tickets actually shows up for their flight. A plane has 100 seats and the airline sold 105 tickets. Find the probability that airline can accommodate all ticketed passenger who shows up. Assume that all passengers act independently.

- 13)** In a certain multiple choice question with 5 possible answers for each question, a student knows the answers of 23 questions out of the total 52 questions. Assume that you pick one of the questions randomly and the student answers it correctly. Find the probability that the student actually knows the answer.
- 14)** A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it.
- a) What is the quality of the parts that make it through the inspection machine and get shipped?
- b) A one-year warranty is given for the parts that are shipped to customers. Suppose that within the first year a good part fails with probability 0.01, while a slightly defective part fails with probability 0.1. What is the probability that a customer receives a part that fails within the first year and therefore is entitled to a warranty replacement?
- 15)** In a city, one half percent of the population has a particular disease. A test is developed for the disease which gives a positive result for those who have and have not disease at 98% and 3% of the time, respectively.
- a) What is the probability that a random person tests positive?
- b) Asim just got the bad news that the test came back positive; what is the probability that Asim has the disease?
- 16)** Consider the game of Let's Make a Deal in which there are three doors, one of which has a car behind it and two of which are empty. You initially select Door 1, then, before it is opened, the host opens one of the other doors, say Door 3, that is empty (selecting at random if both are empty). You are then given the option to switch your selection from Door 1 to the unopened Door 2. What is the probability that you will win the car if you switch your door selection to Door 2? Also, compute the probability that you will win the car if you do not switch. (What would you do?)
- 17)** A factory is manufacturing bolts using three machines,  $A$ ,  $B$  and  $C$ . Of the total output, machine  $A$  is responsible for 25%, machine  $B$  for 35% and machine  $C$  for the rest. It is known from previous experience with the machines that 5% of the output from machine  $A$  is defective, 4% from machine  $B$  and 2% from machine  $C$ . A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from machine  $A$ .
- 18)** A certain town has two taxi companies: Green Birds, whose cabs are green, and Night Owls, whose cabs are black. Green Birds has 15 taxis in its fleet, and Night Owls has 75. Late one night, there is a hit-and-run accident involving a taxi. The town's 90 taxis were all on the streets at the time of the accident. A witness saw the accident and claims that a green taxi was involved. At the request of the police, the witness undergoes a vision test under conditions similar to those on the night in question. Presented repeatedly with a green taxi and a black taxi, in random order, he shows he can successfully identify the color of the taxi 4 times out of 5. Which company is more likely to have been involved in the accident?
- 19)** In front of you are two identical-looking coins. One is fair, and the other biased one comes up 75% Heads of the time. You choose one of the coins, and flip it three times, yielding the sequence HHT. What is the probability that the coin you've been flipping is the unfair one?
- 20)** A blind frog sits on the second of three stones in a line. At each minute, it jumps either to left (with probability 0.4) or right. To the right of third stone there is a lake in which an alligator is waiting for the frog. To the left of the first stone there is a stork waiting for the frog. Find the probability that the stork eats the frog.
- 21)** In a box you have 25 white marbles and 50 black marbles. You also have a lot of black marbles outside the box. Remove two marbles, randomly, from the box. If they are of different colors, put the white one back in the box. If they are the same color, take them out and put a black

marble back in the box. Continue this until only one marble remains in the box. Find the probability that the last marble left in the box is white.

- 22)** You are a prisoner sentenced to death. The Emperor offers you a chance to live by playing a simple game. He gives you 50 black marbles, 50 white marbles and 2 empty bowls. He then says, "Divide these 100 marbles into these 2 bowls. You can divide them any way you like as long as you use all the marbles. Then I will blindfold you and mix the bowls around. You then can choose one bowl and remove ONE marble. If the marble is WHITE you will live, but if the marble is BLACK... you will die." How do you divide the marbles up so that you have the greatest probability of choosing a WHITE marble?
- 23)** A mathematician, his wife, and their son all play chess. One day when the son asked his father some pocket money for Saturday, his father replied, "Let's do it this way. Today is Wednesday. You will play a game of chess tonight, tomorrow, and a third on Friday. If you win two games in a row, you get the money." "Whom do I play first, you or mom?"

"You may have your choice," said the father.

The son knew that his father played a stronger game than his mother. To maximize his chance of winning two games in succession, should he play father-mother-father or mother-father-mother?

- 24)** In a kingdom a man was found guilty and has been sentenced to death. According to law, two shots will to be taken at him from close range. Two bullets are placed into a six-chambered revolver in successive order. He will spin the chamber, close it, and take one shot. If he is still alive, he will then either take another shot, or spin the chamber again before shooting. If the first shot is blank, should he immediately pull the trigger for the second shot or spin the chamber before pulling the trigger?