

METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 1

- 1) In how many ways can you arrange 7 different books, so that a specific book is on the third place?
- 2) In how many ways can you take 3 marbles out of a box with 15 different marbles?
- 3) In a firm there are 20 workmen and 10 employees. In how many ways can you have a committee with 3 workmen and 2 employees?
- 4) In how many ways can you take 5 cards, with at least 2 aces, out of a deck of 52 cards?
- 5) In how many ways can you split a group of 13 students in 3 students and 10 students?
- 6) How many diagonals are there in a convex n -polygon?
- 7) How many 3-digit integers can be written with the digits 0,1, 2, 3, 4?
- 8) Let A be a set with 15 elements. How many subsets of A has 7 or less elements?
- 9) Calculate the term with x^2 in the expansion of $\left(x^3 + \frac{1}{2x}\right)^{10}$.
- 10) In how many ways can you share m identical stones into k boxes so that each pile has at least one stone in it. Assume that boxes labeled with integers $1, 2, \dots, k$.
- 11) How many strictly positive integer solutions (x, y, z) are there, such that $x + y + z = 100$.
- 12) How many terms are there in $(a + b + c)^{20}$?
- 13) A city police department has ten detectives: seven males and three females. In how many ways can a team of three detectives can be chosen to work on a case if
 - a) there are no restrictions?
 - b) the team must have at least one male and one female?
- 14) Given 11 different mathematics and seven different psychology books. How many ways are there for Ahmet to choose a mathematics book, and then for Bekir to choose a mathematics book or a psychology book, and then for Mustafa to chose both a mathematics book and a psychology book.
- 15) How many eight-letter sequences can be constructed using the 26 letters of the English alphabet which
 - a) contain exactly three a's,
 - b) contain three or four vowels {a,e,i,o,u},
 - c) have no repeated letters,
 - d) contain an even number of e's.
- 16) In a group of scientists there are three mathematicians, four physicians, five biologists and six chemists. A committee of four scientists is to be formed. In how many different ways such a committee can be formed if it must contain
 - a) exactly two mathematicians and one physicians.
 - b) one scientist of each field.
 - c) not all scientists from the same field.
 - d) at least two scientists that are not biologists.
- 17) In how many ways can the 26 letters of the English alphabet be placed

- a) in a row?
 - b) in a row so that no two vowels are consecutive?
 - c) in a circle?
 - d) in a circle so that no two vowels are consecutive?
- 18)** Eight students and four faculty members must be seated at two (identical) six-person round tables for lunch.
- a) In how many ways can this be done?
 - b) In how many ways can this be done with at least one faculty member at each table?
- 19)** How many ways are there to distribute 20 different homework problems to ten students if
- a) there are no restrictions?
 - b) each student gets two problems?
- 20)** How many different positive integers can be formed using the digits
- a) 1,2,3,4,5,6,7,8,9
 - b) 1,1,3,3,5,5,7,7,9
 - c) 3,3,3,6,6,6,9,9,9
 - d) 8,8,9,9,9,9,9,9,9
- 21)** A local dairy offers 16 flavors of ice cream.
- a) How many ways are there to purchase eight scoops of ice cream?
 - b) How many ways are there to purchase eight scoops of ice cream of eight different flavors?
 - c) How many ways are there to distribute the eight scoops of part b) to four students so that each student gets two scoops?
 - d) How many ways are there to distribute eight scoops of chocolate ice cream to four students so that each student gets at least one scoop?
 - e) If Zeynep wants to purchase a triple-scoop ice cream cone, how many choices does she have? (Does the order of flavors on the cone matter? Answer the question for both cases.)
- 22)** Consider choosing three digits from $\{0,1,2,3,4,5,6,7,8,9\}$.
- a) In how many ways can this be done?
 - b) In how many ways can this be done if no two of the digits chosen are consecutive?
- 23)** Given five calculus books, three linear algebra books, and two number theory books (all distinct), how many ways are there to
- a) select three books, one in each subject?
 - b) make a row of three books?
 - c) make a row of three books, with one book in each subject?
 - d) make a row of three books, with exactly two of the subjects represented.
 - e) make a row of three books, all in the same subject?
- 24)** If the numbers from 1 to 100,000 are listed, how many times does the digit 5 appear?
- 25)** A sequence of characters is called a palindrome if it reads the same way forward or backward. For example 5FTTF5 is a six-character palindrome, and H8G8H is a five-character palindrome. Some other instances of palindromes: NEVER ODD OR EVEN, ANASTAS MUM SATSANA, RUHU BULUR ULU BUHUR, TOO HOT TO HOOT. Find the number of nine-character palindromes that can be formed using the 29 letters of the Turkish alphabet such that no letter appears more than twice in each of them.

- 26) There are n married couples in a group. Find the number of ways of selecting a woman and a man who is not her husband.
- 27) There are three bridges connecting two towns, A and B . Between towns B and C there are four bridges. A salesperson has to travel from A to C via B . Find
- a) The number of possible choices of bridges from A to C .
 - b) The number of choices for a round-trip travel from A to C .
 - c) The number of choices for a round-trip travel if no bridge is repeated.
- 28) Four station wagons, five sedans, and six vans are to be parked in a row of 15 parking spots. Find the number of ways of parking these vehicles such that
- a) The station wagons are parked at the beginning, then the sedans, and then the vans.
 - b) Vehicles of the same type are parked en bloc.
- 29) Six girls and six boys are to be assigned to stand around a circular fountain. Find the number of such assignments if on either side of a boy there is a girl.
- 30) Find the number of ways of
- a) Assigning 9 students to 11 rooms (numbered serially from 100 to 110) in a dormitory so that each room has at most one occupant.
 - b) Installing nine color telephones (two red, three white, and four blue) in these rooms so that each room has at most one telephone.
- 31) There are four women and nine men in the mathematics department of a university. Find the number of ways of forming a hiring committee consisting of 2 women and 3 men from the department.
- 32) Find the number of ways of seating r people from a group of n people around a round table.
- 33) Find the number of ways of seating 14 people such that 8 of them are around a round table and the rest are on a bench.
- 34) Find the number of ways in which the letters that appear in MISSISSIPPI can be rearranged so that no two S's are adjacent.
- 35) Let X be the set of all words of length 10 in which the letter P appears 2 times, Q appears 3 times, and R appears 4 times. Find the cardinality of X . (Consider the English alphabet.)
- 36) A class consists of 10 mathematics majors and 12 computer science majors. A team of 12 has to be selected from this class. Find the number of ways of selecting a team if
- a) The team has 6 from each discipline.
 - b) The team has a majority of computer science majors.