Answers (Week 1)

1) Remove the third book. Now you can arbitrarily arrange the 6 books in 6! ways.

2) Taking a subset of three elements out a set of 15 elements can be done in $C(15,3)$ ways.

3) First we choose the 3 workmen. This can be done in $C(20,3)$ ways. Then we choose the 2 employees. This can be done in $C(10,2)$ ways. The committee can be assembled in $C(20,3)\cdot C(10,2)$ ways.

4) First consider 5 cards, with exactly 2 aces. For the two aces, we have $C(4,2)$ possibilities and for the three other cards we have $C(48,3)$ possibilities. Then, 5 cards with two aces can be chosen in $C(4,2)\cdot C(48,3)$ ways. Analogously, 5 cards, with exactly 3 aces can be chosen in $C(4,3).C(48,2)$ ways. 5 cards, with exactly 4 aces can be chosen in $C(4,4)\cdot C(48,1)$ ways. So there is a total of $C(4,2)\cdot C(48,3)+C(4,3)\cdot C(48,2)+C(4,4)\cdot C(48,1)$ ways.

5) It is sufficient to choose 3 persons to split the group. This can be done in $C(13,3)$ ways.

6) From each angular point we can count $(n-3)$ diagonals and since there are n points, we have counted $n\cdot(n-3)$ diagonals. But now we have counted twice each diagonal. Hence, there are $\frac{n(n-3)}{2}$ diagonals.

Alternative solution: There are $\binom{n}{2}$ ways to choose a pair of vertices. A pair of vertices describe either a diagonal or a side of the polygon. Since the number of sides is n, the remaining $\binom{n}{2}-n=\frac{n(n-3)}{2}$ pairs correspond to diagonals.

7) There are 4 possibilities for the first figure of the number. There are 5 possibilities for the second figure of the number. There are 5 possibilities for the third figure of the number. So there is a total of $4\cdot 5\cdot 5$ possibilities.

8) Total possibilities = 2^{14}.

9) Applying the binomial theorem we find 1/128.

10) By removing one stone from each pile, this is the number of ways you can arrange $m-k$ identical stones into k (possibly empty) piles. Now, view the k piles as a numbered set. Write on each stone the number of a chosen pile and order the stones accordingly. The numbered stones constitute a combination with repetition of k elements (the numbers) choose $m-k$ (the stones). This can be done in $C(m-1,m-k)$ ways.

11) This is the same problem as 'In how many ways can you arrange 100 identical stones into 3 piles so that each pile has at least 1 stone in it'. From previous problem the answer is $C(99,97) = 4851$ ways.

12) All terms can be written as $A\cdot a^p b^q c^r$ with $p+q+r=20$. The number of terms is the number of solutions of the equation $p+q+r=20$ with p,q,r as positive integer unknowns. Now regard (p,q,r) as three ordered elements. Point 20 times one of these elements, and order these elements in the same order as the given elements. This corresponds with one solution of $p+q+r=20$ and it is a combination with repetition of 3 elements choose 20. The number of terms is the number of such combinations: $C(22,2) = 231$.

13) a) $\binom{10}{3}120$ b) $\binom{7}{2}\binom{3}{1}+\binom{7}{1}\binom{3}{2}=21\times 3+7\times 3=84$

14) $11\,(10\times 9\times 7+7\times 10\times 6)=11{,}550$ (or $11\times(10\times 7)\times 15=11{,}550$)

15) a) $\binom{8}{3}\times 25^5$ b) $\binom{8}{3}5^3 21^5+\binom{8}{4}5^4 21^4$ c) $P(26,8)$ d) $25^8+\binom{8}{2}25^6+\binom{8}{4}25^4+\binom{8}{6}25^2+1$

16) a) $\binom{3}{2}\binom{4}{1}\binom{11}{1}=132$ b) $\binom{3}{1}\binom{4}{1}\binom{5}{1}\binom{6}{1}=360$
c) $\binom{18}{4}-\left(\binom{4}{4}+\binom{5}{4}+\binom{6}{4}\right)=3039$
d) $\binom{13}{4}\binom{5}{0}+\binom{13}{3}\binom{5}{1}+\binom{13}{2}\binom{5}{2}=2925$

17) a) 26! b) $\binom{22}{5}\,21!\,5!$ c) 25! d) $\binom{21}{5}20!\ 5!$

18) a) $\binom{12}{6}\frac{(5!)^2}{2}$ b) $\binom{12}{6}\frac{(5!)^2}{2}-\binom{8}{2}\frac{(5!)^2}{2}$

19) a) $(2^{20})^{10}$ b) $\binom{20}{2}^{10}$

20) a) 9! b) $\frac{9!}{2!^2}$ c) $\frac{9!}{3!^3}$ d) $\frac{9!}{2!5!}$

21) a) $\binom{23}{8}$ b) $\binom{16}{8}$ c) $\frac{8!}{2^4}$ d) 35 e) If order matters, $16^3=4{,}096$; otherwise, $\binom{18}{3}=816$

22) a) $\binom{10}{3}$ b) $\binom{8}{3}$

23) a) 30 b) 720 c) 180 d) 474 e) 66

24) 50,000

25) 29 × 28 × 27 × 26 × 25

26) $n(n-1)$

27) a) 12 b) 144 c) 72

28) a) (4!)(5!)(6!) b) (3!)(4!)(5!)(6!)

29) $5!\,6!=86{,}400$

30) a) $P(11,9)$ b) $\frac{11!}{(2!)(2!)(3!)(4!)}$

31) $\binom{4}{2}\binom{9}{3}=504$

32) $\binom{n}{r}\,(r-1)!$

33) $\binom{14}{8}\,7!\ 6!$

34) $\binom{8}{4}\ \frac{7!}{(4!)(2!)}$

35) $\binom{23}{1}\frac{10!}{(4!)(3!)(2!)}$

36) a) $\binom{10}{6}\binom{12}{6}$ b) $\binom{12}{7}\binom{10}{5}+\binom{12}{8}\binom{10}{4}+\binom{12}{9}\binom{10}{3}+\binom{12}{10}\binom{10}{2}+\binom{12}{11}\binom{10}{1}+\binom{12}{12}\binom{10}{10}$