METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 2

1) Using the letters of English alphabet, in how many different ways is it possible to write a 7 letter string so that
   a) no two letters are the same,
   b) no two consecutive letters are the same,
   c) three letters are the same, the remaining four letters are all different,
   d) if the first letter is a vowel, the last letters is a consonant,
   e) letters are alternatingly consonant / vowel
   f) a consonant is always followed by a vowel,
   g) a letter is used at most twice.

2) In the following pile of letters we start from one of S's and at each step we move to one of the adjacent letters to trace the word STRAMBOŞE. In how many different ways can this be done?

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                          S
                      S   T   S
                  S   T   R   T   S
              S   T   R   A   R   T   S
          S   T   R   A   M   A   R   T   S
      S   T   R   A   M   B   M   A   R   T   S
  S   T   R   A   M   B   O   B   M   A   R   T   S
S   T   R   A   M   B   O   Ş   O   B   M   A   R   T   S
S   T   R   A   M   B   O   Ş   E   Ş   O   B   M   A   R   T   S
```

3) All digits of the integer 6105293748 are distinct and the difference (by absolute value) of the last and first digits is 2. Find the number of all such 10 digit positive integers.

4) There are blue, yellow and white pencils. In how many different ways is it possible to choose 20 pencils if the number of blue pencils has to be even.

5) There are blue, yellow and white pencils. In how many different ways is it possible to choose 15 pencils if the number of blue pencils has to be even.

6) In how many different ways can two children share $2n$ blue, $2n$ yellow and $2n$ white pencils so that each one receives $3n$ pencils?

7) In how many different ways can 6 children share 20 balls so that each one receives
   a) an even number of balls,
   b) an odd number of balls.

8) On a $p \times q$ rectangular grid, a 'right-up-down path' is a path which joins the lower left corner to the upper right corner and at each vertex which moves towards right or up or down. Find the number of right-up-down paths. Below figure illustrates such a path on a 7×8 grid.
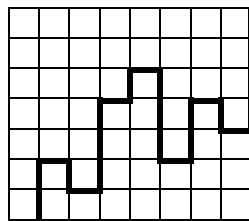
9) A set with n elements has 2^n subsets. Find the sum of cardinalities of all these subsets.

10) A group of 12 students consists of 6 pairs of twins. If a student is not allowed to be in the same group with her twin, in how many different ways can they be partitioned into
a) two equal sized groups?
b) three equal sized groups?

11) A multiple choice consists of 8 questions, with 4 choices for each question. By re-ordering the questions and choices, is it possible to design a different exam paper for each of 1.000.000 students?

12) A question paper consists of 10 questions divided into two parts A and B. Each part contains five questions. A candidate is required to attempt six questions in all of which at least 2 should be from part A and at least 2 from part B. In how many ways can the candidate select the questions if he can answer all questions equally well?

13) The letters of the word NİKSAR are written in all possible orders and these words are sorted as in a dictionary. Find the rank of the word NİKSAR?

14) The letters of the word MALZEME are written in all possible orders and these words are sorted as in a dictionary. Find the rank of the word MALZEME?

15) 15 boys and 6 girls have to stand in a line such that no two girls are next to each other. How many possible ways ?

16) Find the number of ways of filling the cells of a 3×3 table with non-negative integers such that the sum of each row and each column is 15. Below, three examples are given.

2	9	4
7	5	3
6	1	8

15	0	0
0	4	11
0	11	4

3	9	3
11	1	3
1	5	9

17) A secure password must contain
- 2 or 3 upper case letters,
- At least 2 lower case characters,
- 1 or 2 numerals,
- 1 or 2 symbols (! + % & ? * - _ ~).
How many secure passwords with 8 characters can be formed?

18) Find the number of binary sequences of length n if each 1 is followed by an odd number of 0's.

19) The binary string `0011010001110` of length 13 has 7 runs: `00 11 0 1 000 111 0`. Find the number of all such strings (i.e. all binary strings of length 13 with 7 runs).

20) Find the number of ways of arranging the following letters around a circle
   a) A B B C C D D E E F F
   b) A A B B C C D D E E F F
   c) A A A B B B

21) I have 5 identical apples, 8 identical oranges and 13 identical bananas. How many different non-empty baskets can I make consisting of
   a) 5 fruits?
   b) 7 fruits?

22) Let $X = \{1, 2, \ldots, n\}$ and $Y = \{1, 2, \ldots, m\}$. Find the number of monotonic (either increasing or decreasing) functions $X \rightarrow Y$.

23) Let $X = \{1, 2, \ldots, n\}$ and $Y = \{1, 2, \ldots, m\}$. Find the number of non-decreasing functions $X \rightarrow Y$.

24) Let $X = \{1, 2, \ldots, n\}$. Find the number of idempotent functions $X \rightarrow X$.

(A function f is said to be idempotent if $f \circ f = f$.)