METU
DEPARTMENT OF MATHEMATICS


Math 112 Discrete Mathematics

Exercises 3

1) Find the number of positive integers not larger than 1000 which are divisible by 3 or 5.

2) Find the number of five-digit combinations from the set {1,2,3,4,5} in which
a) Some digit appears at least three times.
b) No digit appears more than twice.

3) Find the number of collections of n letters chosen from the set {t, w, x, y, z} if each element appears at least once.

4) 100 students are examined in 5 different courses. In each course, 25 students failed. For any two courses, the number of students failed in both of them is 12. For any three courses, the number of students failed in all three of them is 9. For any four courses, the number of students failed in all four of them is 7. Five students failed in all of the courses. How many students passed all courses?

5) In how many different ways is it possible to rearrange the letters of MATHEMATICS so that no two adjacent letters are the same?

6) Four couples are sitting in a row. Find the number of arrangements in which no person is sitting next to his or her partner. What if they are sitting in a circle?

7) Find the number of all positive integers less than or equal to 1000 that are not divisible by 7, 10, or 15.

8) Find number of non-negative integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 15$ such that $x_i \leq 4$.

9) In the permutation BMTGJSAYIVÇFDOĞRUZHNÜŞEKİLCÖP of Turkish alphabet, the words SAYI, ŞEKİL and DOĞRU can be read directly: BMTGJ<u>SAYI</u>VÇF<u>DOĞRU</u>ZHNÜ<u>ŞEKİL</u>CÖP. Find the number of permutations of the alphabet in which none of these words can be read directly.

10) For the following 7×10 grid, find the number of 'right-up' paths which join the corner A to the corner B.
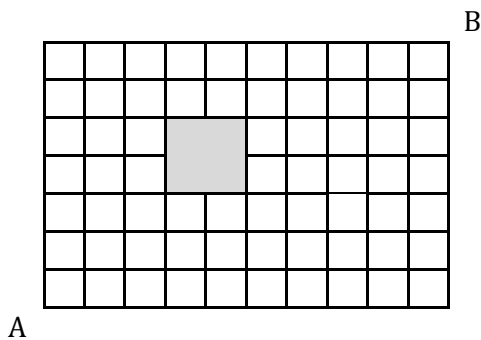
11) For each of the following 7×10 grids, find the number of 'right-up' paths which join the corner A to the corner B.
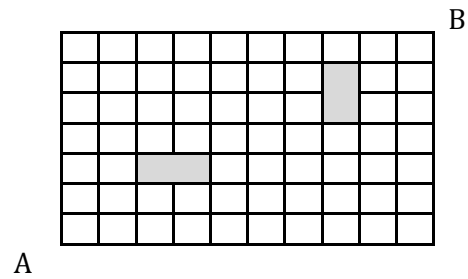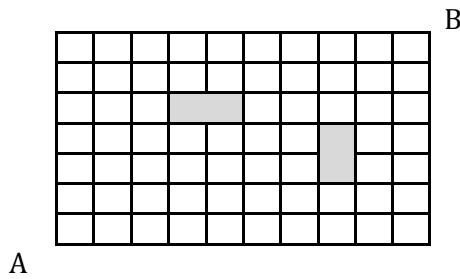
12) Find the number of ways of distributing 20 candies to 5 children so that
a) each of the first two children receives no more than 4 candies,
b) no child receives more than 6 candies.

13) In a permutation $\sigma_1 \sigma_2 \cdots \sigma_n$ of the integers $1, 2, \ldots, n$, the term σ_i is called a fixed point if $\sigma_i = i$. For example, the permutation 4135762 has two fixed points 41<u>3</u>57<u>6</u>2. A permutation without any fixed point is called a derangement. The number of derangements is denoted by D_n. Find the number of
a) derangements,
b) permutations with exactly one fixed point,
c) permutations with exactly k fixed points.

14) How many integers in the set $\{1, 2, 3, 4, \ldots, 360\}$ have at least one prime divisor in common with 360?

15) Let X be a set with m elements and Y be a set with n elements. Show that the number of onto functions from X to Y is

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} (n-k)^m.$$

16) Let X be a set with m elements and Y be a set with n elements. Find the number of functions $f: X \to Y$ such that $|f(x)| = k$.