METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 4

1) Find the number of ways of forming a bouquet of 7 flowers, using roses, cloves, tulips and chrysanthemums.

2) Find the number of ways of distributing 15 candies to 7 children so that the first child recives at most one candy, the second child recives at most two candies,... and the seventh child receives at most seven candies.

3) Find the number of ways of distributing 60 candies to 7 children so that the first child recives at least one candy, the second child recives at least two candies,... and the seventh child receives at least seven candies.

4) Find the number of ways of distributing 30 candies to 7 children so that the first and the second children receive the same number of candies.

5) Find the number of ways of assigning 30 project titles to 7 students.

6) Find the number of ways of assigning 30 project titles to 7 students so that each project is assigned to exactly one student.

7) Find the number of ways of assigning 30 project titles to 7 students so that each student receives at least one project.

8) Find the number of ways of assigning 30 project titles to 7 students so that each project is assigned to at least one student.

9) Find the number of ways of assigning 30 project titles to 7 students so that each project is assigned to at least one student and each student receives at least one project.

10) Find the number of ways of assigning 30 project titles to 7 students so that each project is assigned to at most 5 students.

11) Find the number of ways of assigning 30 project titles to 7 students so that each project is assigned to exactly two students and each student receives at least one project.

12) How many ways can n married couples be paired up to form n couples so that each couple consists of a man and a woman and so that no couple is one of the original married couples?

13) Find the number of k −subsets of $\{1,2,\dots,n\}$ which do not contain any pair of consecutive integers.

14) Let $X=\{1,2,3,4,5\}$ and $Y=\{1,2,3,4,5,6,7,8\}$. Find the number of functions $f: X \to Y$ if
a) $f(i) \leq i$ for all $i \in X$,
b) f is one-to-one and $f(i) \leq i$ for all $i \in X$.

15) Let $X=\{1,2,3,4,5\}$ and $Y=\{1,2,3,4,5\}$. Find the number of functions $f: X \to Y$ if
c) $f(i) \neq i$ for all $i \in X$,
d) f is one-to-one and $f(i) \neq i$ for all $i \in X$.

16) Find the number of words of length 10, consisting of letters a, b, c where a b is followed by two c's.

17) A class consists of 10 boys and 10 girls. Find the number of ways of splitting them into 10 equal sized groups

a) if each group consists of either two girls or two boys,
b) if each group consists of a boy and a girl,
c) if there are exactly 6 mixed (consisting of a boy and a girl) groups.

18) Consider a chess board of dimension $p \times q$ (p rows, q columns). Find the number of placing n checkers on the board such that each cell (unit square) contains at most one checker.

a) If n is not known.
b) If n is fixed.
c) If $n = p$ and on each row there is exactly one checker.
d) If $n = p, p \leq q$ and on each row there is exactly one, on each column there is at most one checker.
e) If $n = p = q$ and on each row and on each column there is exactly one checker.
f) If $n = p, p \geq q$ and on each row there is exactly one, on each column there is at least one checker.
g) If $n = 2p$ and on each row there are exactly two checkers.

19) In the spring semester, four elective courses are offered: Math 307, Math 369, Math 404 and Math 427. Each course completed the registration period with full capacity of 40 students. It is observed that 4 students are registered to all of the four courses. 6 students registered to 307, 404 and 427; and there are 7 students registered to any other triple of these courses. 17 students registered to 307 and 427; 15 students are registered to 369 and 404, and for any other pair of these courses there are 16 students in common.

a) Find the number of students which registered to at least one of these courses.
b) Find the number of students which registered to exactly one of these courses.
c) Find the number of students which registered to exactly two of these courses.
d) Find the number of students which registered to only Math 307.