METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 5

1) In a bag there are 5 yellow balls labeled y_1, y_2, y_3, y_4, y_5 and 4 blue balls labeled b_1, b_2, b_3, b_4. First a ball is drawn randomly and put in a box, then a second ball and then a third ball is drawn and put in the same box.
a) For three balls in the box, how many diffferent configurations are there?
b) In how many configurations there are three yellow balls? A yellow and two blue balls? Two yellow balls and a blue ball? Three blue balls?
c) If three balls are drawn at once, instead of being picked one by one, how would you answers the above parts?

2) In a bag there are 5 yellow and 4 blue balls. All the balls, except their colors, are identical. Three balls are drawn and put in a box.
a) For three balls in the box, how many diffferent configurations are there?
b) In how many configurations there are three yellow balls? A yellow and two blue balls? Two yellow balls and a blue ball? Three blue balls?

3) Let $X = \{1,2,3,\dots,30\}$ and $Y = \{1,2,3,\dots,10\}$. Find the number of functions $f: X \to Y$ such that for any $y \in Y$, $|\{x \in X | f(x) = y\}| = 3$.

4) Let A be a finite set with $|A| = n$.
a) Find the number of reflexive relations defined on A.
b) Find the number of symmetric relations defined on A.
c) Find the number of reflexive and symmetric relations defined on A.
d) Find the number of anti-symmetric relations defined on A.

5) Let $X = \{1,2,3,\dots,15\}$ and $Y = \{1,2,3,4,5\}$. Find the number of functions $f: X \to Y$ such that for any $y \in Y$, $|\{x \in X | f(x) = y\}| = y$.

6) Find the number of ways of arranging 5 blue and 10 yellow balls in a row so that
a) there is at least one ball between any two blue balls,
b) there are at least two balls between any two blue balls.

7) Find the number of ways of arranging 5 blue, 5 yellow and 5 white balls in a row so that
a) there is at least one ball between any two blue balls,
b) there are at least two balls between any two blue balls.

8) Find the number of distributing 5 orange candies and 5 lemon candies to 2 children.

9) Find the number of distributing 5 orange candies and 5 lemon candies to 2 children so that each child receives at least one candy of each kind.

10) In how many different ways can 11 schoolboys be separated in 4 groups of different sizes?

11) In how many different ways can 13 schoolboys be separated in 4 groups of different sizes?