METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 6

1) Five girls travel with one boy to a math contest. They have four hotel rooms, numbered 1 through 4. Each room can hold up to two people, and the boy has to have a room to himself. How many different ways are there to assign the students to the rooms?

2) 30 students has to share 10 hotel rooms numbered 1501 through 1510. Find the number of ways of assigning the students to the rooms if
a) each room can hold at most three people,
b) room 1501 can hold 4 people and each of the remaining rooms can hold three people.

3) Determine the number of ways to distribute 10 orange drinks, 1 lemon drink, and 1 ayran to four thirsty students such so that each student gets at least one drink, and the lemon drink and ayran go to different students.

4) Find the number of ways of distributing 25 candies to 6 children such exactly half of the boys receive an even number of candies each.

How many triangles appear in the diagram? For which arrangement of three overlapping triangles the number of triangles that appear in the diagram is greatest possible?

5) Using the letters of , KASTAMONU how many 4-letter words can be written?

6) Given a cube with vertices $A_1, A_2, \dots, A_8$. How many triangles are there with each vertex being a vertex of the cube? How many of these triangles are equilateral?

7) A circular table has exactly 60 chairs around it. There are N people seated around this table in such a way that the next person to be seated must sit next to someone. What is smallest possible value of N?

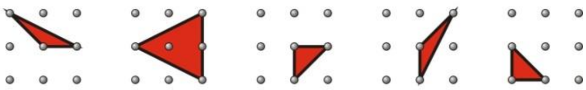
8) 4 girls and 8 boys are standing together.
a) In how many ways can they stand around a circle?
b) In how many ways can a 7-person committee be selected from the group if at least 2 girls must be included?

9) Positive integers are partitioned as follows.

(1) (2 3) (4 5 6) (7 8 9 10) (11 12 13 14 15) (16 17 18 19 20 21) (22 23 24 25 26 27 28)

Each part contains one more integer than the preceeding part. For example, there are 6 integers in part 6 and their sum is 111. What is the smallest integer in part 30? What is the sum of integers in part 30?

10) 9 points are arranged to form a 3×3 square lattice. How many different triangles can be drawn with each vertex at one of the lattice points? The figure shows 5 such triangles.



How many non-congruent triangles can be drawn? In the above figure, the first (counting from left) and the fourth triangles are congruent, third and fifth triangles are also congruent. Thus, the figure contains 3 non-congruent triangles.

11) There are five partitions of 8 with the largest part 4:

$$4+4, \quad 4+3+1, \quad 4+2+2, \quad 4+2+1+1, \quad 4+1+1+1+1$$

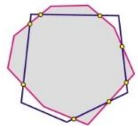
The number of partitions into 4 parts is also five:

$$2+2+2+2, \quad 3+2+2+1, \quad 3+3+1+1, \quad 4+2+1+1, \quad 5+1+1+1$$

Show that for any integer n, the number of partitions with k parts is equal to the number of partitions with the largest part k.

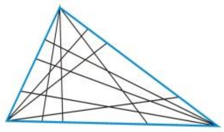
12) Given a circle with n points marked on it. Each pair of points are joined by a chord such that no three chords intersect in a common point. Find the number of triangles formed in the circle.

13) A convex polygon with n_1 sides overlaps with another polygon with n_2 sides. What is the largest possible number of the intersection points of their boundaries?



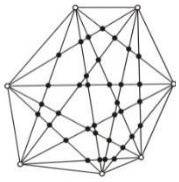
14) You must arrange 3 oak trees, 4 maple trees and 5 birch trees in a line such that no two of the birch trees are adjacent to one another. In how many ways can you arrange the trees? (Assume that two oak trees are indistinguishable, as are two maples and two birches.

15) On each edge of a triangle n points are marked and each point is joined to the opposite vertex by a stright line. If no three points intersect at a point, how many regions are formed in the triangle?



16) How many different 10-digit numbers can be formed from the digits 1, 2, 3 where digit 3 in each number is found exactly three times ? How many of those numbers can be divided by 6 ?

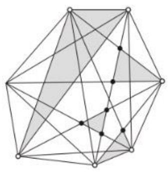
17) No three diagonals of a convex n −gon intersect at a point. Find the number of intersection points of these diagonals in the n −gon.



18) Find the number of positive integers not exceeding 1000 that are neither the square nor the cube of a positive integer.

19) Given a convex n −gon $A_1, A_2, \dots, A_n$. Find the number of ways of chosing k of these vertices which define a convex k −gons with no side common with the n −gon.

20) No three diagonals of a convex n −gon intersect at a point. Find the number of all triangles with vertices at the intersection points of the diagonals.



21) Find the number of ways of arranging the integers 1,2,3, … ,20 in a 2×10 array such that each row is in ascending order and in each column, integer in the first row is larger than the integer in the second row. An eaxmple of such an array is given below.

4	5	6	8	10	13	14	16	18	20
1	2	3	7	9	11	12	15	17	19

22) There are 15 members of a club. To open a room at least 8 members must be present to unlock the room. No group of 7 or fewer can open the room. Each lock has a different key, but you can make several copies of the same key to disbribute to club members. What is the fewest number of locks and keys that you will need and how would the keys be distributed among members?