Math 112 Discrete Mathematics

Exercises 8

1) When climbing a staircase, with each step she takes Ayşe, moves up either one stair or two stairs. Find the number of different ways Ayşe can climb the staircase which consists of n stairs.

2) Find the number of subsets of $\{1,2,3,\dots,n\}$ which do not contain any pair of successive integers.

3) Find the number of ways of tiling a $1 \times n$ rectangular board using 1×2 and 1×1 pieces.

4) Find the number of ways of tiling a $2 \times n$ rectangular board using 1×2 and 2×2 pieces.

5) How many different messages can be transmitted in n microseconds using three different signals if one signal requires 1 microsecond for transmittal, the other two signals require 2 microseconds each for transmittal, and a signal in a message is followed immediately by the next signal?

6) Find the number of permutations of $\{1,2,3,\dots,n\}$ in which no integer is moved more than one place from its original position.

7) Find the number of strings of length n formed with letters A, B and C if the number of A's is even.

8) Find the number of strings of length n formed with letters A, B and C that do not contain AA.

9) Find the number of strings of upper case letters of length n which contain an even number of Z's.

10) Find the number of strings of upper case letters of length n that do not contain ZZ.

11) Find the largest possible number of regions in the plane that can be defined by

a) n straight lines in the plane.

b) n circles in the plane.

c) n triangles in the plane.

d) n rectangles in the plane.

12) A circular disk is separated into n sectors by n radii. Find the number of ways of painting each sector in blue, red or white such that no two neighboring sectors are of the same color.

13) Each unit square of a 2×20 chessboard is to be painted in blue, yellow or red. Find the number of possible ways if

a) two adjacent squares cannot receive the same color,

b) two red squares are not allowed to be adjacent.