METU
DEPARTMENT OF MATHEMATICS

Math 112 Discrete Mathematics

Exercises 9

1) Let $A = \{1,2,3,4,5,6,7,8\}$.
a) If five integers are selected from A, must at least one pair of integers have a sum 9?
b) If four integers are selected from A, must at least one pair of integers have a sum 9?

2) Given a group of n women and their husbands, how many people must be chosen from this group of $2n$ people in order to guarantee that the set of those selected contains a married couple

3) In a round-robin tournament (in which every player plays against every other player exactly once), suppose that each player wins at least once. Show that there are at least two players with the same number of wins

4) To guarantee that there are ten diplomats from the same continent at a party, how many diplomats must be invited if they are chosen from
a) 12 Australian, 14 African, 15 Asian, 16 European, 18 South American, and 20 North American diplomats.
b) 7 Australian, 14 African, 8 Asian, 16 European, 18 South American, and 20 North American diplomats.

5) Show that, in a group of 150 people, at least six must have the same last initial.

6) There are 42 students who are to share 12 computers. Each student uses exactly one computer and no computer is used by more than 6 students. Show that at least five computers are used by three or more students.

7) A bag contains exactly 6 red, 5 white, and 7 marbles. Find the least number of marbles to be picked which will ensure that either at least 3 red or at least 4 white or at least 5 blue marbles picked.

8) In any set of 1001 integers chosen from $\{1,2,3,\dots,2000\}$, show that there must be two members such that one is divisible by the other.

9) Suppose that the numbers $1,2,\dots,100$ are randomly placed in 100 locations on a circle. Show that there exist three consecutive locations so that the sum of integers at these locations is at least 152.

10) A violinist practiced for a total of 110 hours over a period of 12 days. Show that he practiced at least 19 hours on some pair of consecutive days. Assume that he practiced a whole number of hours on each day.

11) Prove that among any five points selected inside an equilateral triangle with side equal to 1, there always exists a pair at the distance not greater than $\frac{1}{2}$.

12) Each of the given 9 lines cuts a given square into two quadrilaterals whose areas are in proportion 2: 3. Prove that at least three of these lines pass through the same point.

13) Five points are chosen at the nodes of a square lattice (grid). Why is it certain that at least one mid-point of a line joining a pair of chosen points, is also a lattice point?

14) Suppose $f(x)$ is a polynomial with integral coefficients. If $f(x) = 2$ for three different integers a, b, and c, prove that, for no integer, $f(x)$ can be equal to 3.

15) Prove that there exist two powers of 3 whose difference is divisible by 1997.

16) Prove that there exists a power of three that ends with 001.

17) If more than 500 integers from $\{1,2,\dots,1000\}$ are selected, then some two of the selected integers have the property that one divides the other.

18) A person takes at least one aspirin a day for 30 days. If he takes 45 aspirin altogether, in some sequence of consecutive days he takes exactly 14 aspirin.

19) A theater club gives 7 plays one season. Five women in the club are each cast in 3 of the plays. Then some play has at least 3 women in its cast.

20) Prove that at any party of n people, some pair of people are friends with the same number of people at the party.

21) Let $S = \{3,4,5,6,7,8,9,10,11,12,13\}$. Suppose six integers are chosen from S. Must there be two integers whose sum is 16?

22) Let $S = \{3,4,5,6,7,8,9,10,11,12,13\}$. Suppose seven integers are chosen from S. Must there be two integers whose sum is 16?

23) Let $S = \{7, 8, 9, \dots, 97\}$. At least how many integers should be chosen from S to guarantee the existence of two integers whose sum is 100?

24) Let A be a set of six positive (distinct) integers each of which is less then 13. Show that there must be two distinct subsets of A sums of whose elements are same.

25) During a campaign, a politician visits 45 towns in 30 days. If he visits a positive whole number of towns each day, show that there must exist some period of consecutive days during which he visits exactly 14 towns.